Little M-theory

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ABSTRACT: Using the language of theory space, i.e. moose models, we develop a unified framework for studying composite Higgs models at the LHC. This framework — denoted little M-theory — is conveniently described by a theoretically consistent three-site moose diagram which implements minimal flavor and isospin violation. By taking different limits of the couplings, one can interpolate between simple group-like and minimal moose-like models with and without T-parity. In this way, little M-theory reveals a large model space for composite Higgs theories. We argue that this framework is suitable as a starting point for a comprehensive study of composite Higgs scenarios. The rich collider phenomenology of this framework is briefly discussed.

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1. Motivation

If there is a natural solution to the hierarchy problem, then the data from the Large Hadron Collider (LHC) will be spectacular. Almost all natural theories predict new colored states around the TeV scale to cancel the quadratically divergent top contribution to the Higgs potential, and at a *pp* collider, there is a large cross section for producing these new states. But while we will certainly know *whether* there is new physics at the TeV scale, it may be more difficult to know *what* new physics we are actually producing. As the commissioning of the LHC draws near, it is important to simulate a large number of models to determine which experimental observables can best distinguish different possibilities for physics beyond the standard model.

Many scenarios for stabilizing the electroweak scale have been proposed. Besides low energy supersymmetry (SUSY), there are various classes of non-SUSY theories in the literature: technicolor [1, 2], top color [3], Higgsless [4], Universal Extra Dimensions (UED) [5, 6], little Higgs [7, 8], holographic Higgs [9, 10], twin Higgs [11], and so on. Within each class, there are a large number of variants that are in principle distinguishable given enough experimental data, and each variant has a set of adjustable parameters. It is challenging task to decide whether any specific model deserves a detailed study of its collider signals. Moreover, it is not clear whether any particular model should be treated as unique in its class or part of a bigger continuous model space.

In this paper, we attempt to develop a unified framework for describing a large class of non-SUSY models, with the goal of mimicking the current situation for supersymmetric theories. Though there are a lot of SUSY-breaking mechanisms which predict different low energy spectra, we have general low energy Lagrangians — i.e. the MSSM [12] and its extensions — whose parameter space interpolates among various SUSY-breaking models. Therefore, at the LHC we need not conduct specific searches for any particular SUSY-breaking mechanism. Rather, we can attempt to measure the parameters of a general Lagrangian, and such measurements will hopefully lead us towards a particular model.

The key observation that makes non-SUSY unified frameworks possible is that the low energy physics of many seemingly dissimilar theories can actually be different limits of the same theory once we integrate out degrees of freedom inaccessible at the LHC. Because these frameworks are most simply described by moose diagrams [13], we call them "M-theories". As we will see, most non-SUSY theories have an associated moose, and different non-SUSY theories are often associated with the *same* moose, so by studying the LHC phenomenology of a single M-theory, one can simultaneously explore many different non-SUSY solutions to the hierarchy problem. Regardless of whether an extra dimension is flat [14] or warped [15, 16], or whether a little Higgs comes from a simple group [17, 18] or a minimal moose [19, 20], there is a single low energy effective description relevant for the LHC. The utility of such a general framework is not just that it interpolates among various known models; it also reveals the existence of a larger model space with richer structure than any of the known limits.

There are two facts — one theoretical, one experimental — that make M-theories relevant for the LHC. On the theory side, it is important that moose diagrams are general enough to approximate the low energy physics of known non-SUSY theories. Almost all non-SUSY theories for physics beyond the standard model are either (a) based on extra dimensions, (b) based on moose diagrams, (c) well-approximated by extra dimensions using the AdS/CFT correspondence [21 – 25], or (d) well-approximated by mooses using the technique of little technicolor [26]. With the help of deconstruction [27, 28], we can indeed combine all four possibilities into M-theories, moose diagrams whose various limits reproduce the important features of different underlying non-SUSY theories.

On the experimental side, it is important that a "low energy" moose description can be justified at the energy scales accessible to the LHC. Otherwise, a moose would fail to capture interesting "high energy" LHC signatures and a more complete theory would be needed. In many non-SUSY models, there is a layer of weakly coupled physics between the

electroweak scale and a cut-off Λ. The hierarchy between M_{Pl} and Λ is either taken care of by some strong dynamics or left to an unspecified UV completion. As long as Λ is heavy enough, the LHC cannot probe the underlying UV model directly, and a weakly coupled low energy effective description will suffice. Though the center of mass energy of the LHC is 14 TeV, the discovery reach is typically smaller by about factor of 3 or 4, so Λ need only be a few TeV to safely use an M-theory approximation. This does mean, however, that M-theories will not be as useful for describing theories like technicolor, where the scale of strong dynamics is expected to be within the reach of the LHC.

A straightforward method to construct a non-SUSY unified framework would be to use a purely bottom-up approach motivated by our UV ignorance. Apart from high energy modes, various non-SUSY models yield largely similar low energy spectra. Therefore, one could simply write down a theory that includes only the layer of new physics states between the electroweak scale and Λ, which usually fill out simple representations of $\text{SU}(2)_L \times \text{U}(1)_Y$. There could be extra broken U(1)s or other extensions of the electroweak gauge symmetries, but with certain assumptions of minimality, it is not hard to write down a generic Lagrangian which parametrizes the interactions of all of those states.

In this study, we seek to go beyond such a bottom-up approach and incorporate lessons from the past several decades of non-SUSY model building. In the standard model, we minimally expect new physics at $\Lambda \sim 4\pi v_{\text{EW}}$ to unitarize W-W scattering. However, if we naively use $4\pi v_{\text{EW}}$ to set the size of higher dimension operators in the standard model, then we find too large corrections to electroweak observables constrained by precision measurements. This tension at the percent-level is known as the little hierarchy problem [29], which has received a lot of attention in the post-LEP era. It is therefore particularly well-motivated to study models that attempt to explain the separation between the electroweak scale and a cut-off Λ at around 10 TeV, the scale of new physics suggested by precision electroweak measurements [30]. Like the bottom-up approach, we start by writing down an effective theory beneath the cut-off scale, but we will try to preserve as many of features of realistic theories as we can in order to focus our attention on preferred regions of model space. In particular, our framework allows for T-parity [31, 32], a $\mathbf{Z}_2$ symmetry that generically protects precision observables.

From the point of view of the little hierarchy, one of the most interesting non-SUSY scenarios to be probed at the LHC is a composite Higgs [33, 34]. Generally speaking, these are models where there is a Goldstone mode with the quantum numbers of a Higgs doublet and where same-statistics partners cancel divergent contributions to the Higgs potential. In composite Higgs theories (as opposed to technicolor or Higgsless theories) there is a range of energies in which W-W scattering is unitarized by a weakly coupled Higgs doublet, and only at a higher scale $\Lambda \sim 4\pi f_{\text{eff}}$ does one see that the doublet is accompanied by additional, possibly strongly coupled, high energy modes. As long as there is a mechanism to guarantee $f_{\text{eff}} \gg v_{\text{EW}}$, then not only will the naïve corrections to precision electroweak observables be suppressed, but the scale Λ will also be beyond the reach of the LHC, justifying an M-theory description.

Composite Higgs theories can be classified according to the quantum numbers of the new heavy modes that regulate higgs-gauge loops. In minimal moose-like theories, the

electroweak gauge group is doubled, yielding massive W' partners. In simple group-like theories, the electroweak gauge group is embedded in a larger gauge group, yielding massive X/Y (off-diagonal) gauge boson partners. For example, most collider studies to date have focused on the littlest Higgs [35], which is a minimal moose-like theory because the fundamental gauge group contains two copies of SU(2). Holographic composite Higgs models are a hybrid scenario with both W' and X/Y states. Here, we construct a "little M-theory" suitable for collider studies that interpolates between both choices for the new heavy spin-1 modes, providing a single phenomenological model where many different LHC signatures can be explored.

While the term "M-theory" [36] suggests the existence of a unique description of non-SUSY LHC physics, there are in fact many different M-theories just as there are many different SUSY extensions of the standard model. In general, there are two orthogonal directions one could explore in the model space of mooses. In this paper, we focus on the different mechanisms one can employ for canceling quadratic divergences, studying various limits of one underlying moose. However, the requirement of a composite Higgs does not fix the symmetry structure and its breaking pattern, so one could also explore models with different global and gauge symmetries. In the spirit of the MSSM, we choose the minimal symmetry structure which still allows for custodial SU(2). We argue that this is a useful framework which contains generic phenomenology. It is straightforward to adjust the symmetry structure of the moose if we experimentally discover more or less exotics.

To summarize, the new physics which will be probed by the LHC can be described two broad categories of models: SUSY theories and theory space theories. Just as the MSSM is a interesting example of a SUSY theory, little M-theory is an interesting moose model that can interpolate between many different ultraviolet models. Though it is indeed possible to deform this little M-theory into a UED or Higgsless model, we will stay in the composite Higgs limit in order to keep the little M-theory Lagrangian as simple as possible. We comment on the implications of such deformations in the conclusion.

In the next section, we review the low energy equivalence between different types of non-SUSY models and present a toy little M-theory that interpolates between three known composite Higgs models based on very different starting assumptions. Readers interested in the actual model can skip directly to section 3 where we present a theoretically consistent little M-theory based on the coset space Sp(4)/SO(4). This moose is suitable for collider studies and has many adjustable parameters to deform the spectrum and decay modes. In section 4, we discuss experimental constraints and preferred region of the parameter space, and describe several familiar limits of little M-theory. Interesting new features of the phenomenology as well as open questions are commented in section 5. Conclusions and the possibility of extending the little M-theory approach to other classes of non-SUSY theories are contained in section 6.

2. Known mooses and little M-theories

As already mentioned, there are two observations that justify the use of little M-theories for describing LHC phenomenology. The first is that different ultraviolet theories can have the

same low energy physics. A classic example of this is a KK tower. If we deconstruct a (non-gravitational) extra dimension [27, 28], then the first n KK modes are well approximated by an n-site moose diagram. We can improve the approximation either by adding sites or by introducing non-local interactions in theory space. In this way, an extra dimensional theory and an n-site moose theory have nearly identical LHC phenomenology as long as only the first n KK modes are kinematically accessible at the LHC. Moreover, both flat and warped extra dimensions can be described by the same n-site moose, the only difference being the values of the gauge couplings and decay constants on the sites and links [37, 38]. In the next subsection, we review low energy equivalences in the context of electroweak physics and show why mooses are a convenient way to encode infrared degrees of freedom.

The second justification for using little M-theories is that if we take the little hierarchy problem seriously, then we expect a hierarchical separation between v_{EW} and f_{eff}. In this way, the scale of possible strong dynamics $\Lambda \sim 4\pi f_{\mathrm{eff}}$ is beyond the reach of the LHC, and a weakly coupled moose description will suffice.[1] From a model building perspective, one would like some symmetry reason to guarantee this little hierarchy. Indeed, the novel structure of the Higgs potential is the *raison d'être* for little Higgs theories, in that these theories exhibit a parametric separation between v_{EW} and f_{eff} compared to generic composite models. From the point of view of the LHC, though, the origin of the Higgs potential has little impact on collider signatures. For example, the minimal moose contains extra link fields and plaquette operators to generate a large enough Higgs quartic without introducing a large Higgs mass, but this extended Higgs sector just introduces new heavy states with no generic pattern. Thus, in both the toy example in section 2.2 and the complete theory in section 3, we will ignore the origin of the Higgs potential and adjust it by hand.

2.1 Mooses and low energy equivalence

Mooses are a simple and flexible language to describe low energy theories. Sites on a moose diagram correspond to the global/gauge symmetries of the theory, and links correspond to fields that transform as fundamentals or anti-fundamentals under the appropriate symmetries. In this paper, most link fields will be non-linear sigma fields, so these descriptions will only be valid up to the scale of Goldstone unitarity violation.

The low energy equivalence is easy to understand in the moose language. The simplest moose relevant for electroweak physics is the standard model with a Higgs boson. Ignoring fermions, color, and hypercharge, the electroweak sector is described by the following moose:

$$\text{Global}: \quad \mathrm{SU}(2)_L \qquad\qquad \mathrm{SU}(2)_R$$

$$\text{Gauged}: \quad \mathrm{SU}(2)_L \tag{2.1}$$

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[1]In the context of both composite Higgs and technicolor theories, there have been attempts to bring the ratio $f_{\mathrm{eff}}/v_{\mathrm{EW}}$ closer to 1 while evading the naïve bounds from precision electroweak measurements (see e.g. [40 – 42]). If strong dynamics is seen at the LHC, then a moose description will act like a "techni-QCD" chiral Lagrangian.

where H can be written in terms of the usual Higgs doublet h as

$$H = \frac{1}{\sqrt{2}} \left(\epsilon\, h \ \ h^\dagger \right) = \frac{1}{\sqrt{2}} \begin{pmatrix} h^0 & h^- \\ -h^+ & h^{0*} \end{pmatrix}, \tag{2.2}$$

and H transforms as $H \to g_L^\dagger H g_R$ under the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ global symmetry. The advantage of the H notation over doublet notation is that eq. (2.1) makes custodial SU(2) symmetry manifest.

Now, it is straightforward to construct many different theories whose low energy physics is well described by eq. (2.1). At energies much below the mass of the physical Higgs boson, we can simply replace H with a non-linear sigma field Σ

$$\Sigma = \frac{v_{\mathrm{EW}}}{2} e^{2i\vec{\pi}\cdot\vec{\sigma}/v_{\mathrm{EW}}}, \tag{2.3}$$

where $\vec{\sigma}$ are the Pauli matrices. In the language of CCWZ [43, 44], Σ describes the Goldstone bosons arising from the spontaneous breakdown of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ to the diagonal $\mathrm{SU}(2)_V$. Important for our purposes, there are many ways to get a non-linear sigma model from a high energy theory. For example, in technicolor the Σ field arises from a fermion condensate

$$\tag{2.4}$$

where beneath Λ_{TC}, we can identify Σ with fluctuations about the condensate $\langle \psi\psi^c \rangle$. We can also generate a non-linear sigma model from a Wilson line in a flat or warped extra dimension. Imposing the appropriate boundary conditions on an interval with a bulk SU(2) gauge fields

$$\tag{2.5}$$

the Wilson line $e^{i\int A_5 dx^5}$ has the same transformation properties as Σ. A particularly interesting extra dimensional geometry is AdS$_5$, and eq. (2.5) is expected to be dual to a quasi-CFT with a gauged $\mathrm{SU}(2)_L$ symmetry that is spontaneously broken in the infrared [24, 25], i.e. the Higgsless dual of technicolor.

We can generate moose diagrams with additional sites with the same light degrees of freedom by deconstructing these extra dimensions. The geometry of the extra dimension

is encoded in the different pion decay constants on the various links [37–39]

$$
\begin{array}{cccccc}
\text{Global}: & \text{SU}(2)_1 & \text{SU}(2)_2 & & \text{SU}(2)_N & \text{SU}(2)_{N+1} \\
& \bigcirc \overset{\Sigma_1}{\underset{\to}{\quad}} \bigcirc \overset{\Sigma_2}{\underset{\to}{\quad}} \cdots \overset{\Sigma_{N-1}}{\underset{\to}{\quad}} \bigcirc \overset{\Sigma_N}{\underset{\to}{\quad}} \bigcirc \\
\text{Gauged}: & \text{SU}(2)_1 & \text{SU}(2)_2 & & \text{SU}(2)_N &
\end{array}
\tag{2.6}
$$

The original Σ field is given by

$$
\Sigma = \Sigma_1 \Sigma_2 \cdots \Sigma_N \tag{2.7}
$$

and we can explicitly recover eq. (2.1) from eq. (2.6) by integrating out sites corresponding to heavy gauge bosons.[2]

Finally, we can use the trick of hidden local symmetry [45] or little technicolor [26] to convert any non-linear sigma model into a moose diagram. Using CCWZ, any spontaneous symmetry breaking pattern can be described in terms of a G/H non-linear sigma model with a subgroup $F \subset G$ weakly gauged. We can then introduce a new "ρ meson" gauge field to generate the moose diagram

$$
\begin{array}{ccc}
\text{Global}: & G & G \\
& \bigcirc \overset{\Sigma}{\underset{\to}{\qquad}} \bigcirc \\
\text{Gauged}: & F & H
\end{array}
\tag{2.8}
$$

The original CCWZ non-linear sigma model can be recovered from eq. (2.8) by integrating out the H gauge bosons. Note that the low energy moose from little technicolor is identical to the two-site deconstruction of the warped AdS_5 dual theory [9].

We have seen that a standard model Higgs, technicolor, extra dimensions, quasi-conformal field theories, and general non-linear sigma models all have descriptions in terms of mooses. In the case of electroweak physics circa 1980, the only relevant moose diagram for discovering the W and Z bosons was

$$
\begin{array}{ccc}
\text{Global}: & \text{SU}(2)_L & \text{SU}(2)_R \\
& \bigcirc \overset{\Sigma}{\underset{\to}{\qquad}} \bigcirc \\
\text{Gauged}: & \text{SU}(2)_L & \text{U}(1)_Y
\end{array}
\tag{2.9}
$$

and until the precision electroweak tests from LEP experiments, in principle we didn't even know whether the W and Z bosons were fundamental gauge fields whose mass came from spontaneous symmetry breaking, or "ρ mesons" from a strongly coupled theory as

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[2]In general, integrating out sites from a moose will induce non-local interactions in theory space because the wave function of heavy gauge bosons span the entire space. In the special case of AdS_5, these non-localities are suppressed because the heavy mode wave functions are localized [26]. In any case, we can always capture the effect of theory space non-locality by introducing new interactions at higher order in the Σ fields.

in the Abbott-Farhi model [46, 47]. Even today, while precision electroweak suggests a physical Higgs boson should exist in the form of a linear sigma model UV completion for the standard model, finely-tuned technicolor and Higgsless theories can still satisfy the experimental bounds [48, 49].

The lesson from looking at different UV completions of the standard model is that moose diagrams are a convenient way to organize one's thinking about non-SUSY physics beyond the standard model. Unlike extra dimensional theories which yield a complete tower of KK modes, one can adjust a moose deconstruction to only include modes relevant for a given collider. Moreover, mooses carry no implicit theoretical biases and merely give a consistent framework to describe the relevant spin-0 and spin-1 degrees of freedom.[3] Of course, the LHC will be able to tell the difference between a physical Higgs and technicolor. However, if we take the little hierarchy problem seriously, then in the context of all current non-SUSY proposals, we expect a non-linear sigma model description to suffice for at least the initial running at the LHC, similar to the status of the standard model in the pre-LEP era.

2.2 A toy little M-theory

Little M-theories are classified according to their symmetry structure and the embedding of the Higgs. Because there are many different symmetry breaking patterns that can yield a doublet charged under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ at low energies, there is no unique M-theory to describe composite Higgs theories. Rather, using the tools of the previous subsection, every composite Higgs theory can be described by a moose diagram, and in certain cases, one can interpolate between different models by taking different limits of the same M-theory. In this subsection, we will show how this interpolation works in a toy little M-theory without hypercharge or fermions.

This toy model is based on the coset space $\mathrm{SU}(3)/\mathrm{SU}(2)$. In particular, imagine a triplet of a global $\mathrm{SU}(3)$ that takes a vacuum expectation value (vev).

$$\Phi = e^{i\Pi/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \tag{2.10}$$

The $\mathrm{SU}(3)/\mathrm{SU}(2)$ goldstone matrix contains a doublet h and a singlet η under the unbroken $\mathrm{SU}(2)$.

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h_1 \\ 0 & 0 & h_2 \\ h_1^\dagger & h_2^\dagger & 0 \end{pmatrix} + \frac{1}{2\sqrt{3}} \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & -2\eta \end{pmatrix} \tag{2.11}$$

There are at least three theories based on this coset space, namely the simple group little Higgs [17], the minimal moose little Higgs [19], and the original holographic Higgs [9]. As we will see, they can all be described by the same three-site M-theory. Further variations are discussed in [26].

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[3]There is no healthy lattice description of gravity, so while mooses can describe heavy spin-2 modes [50], there is no straightforward extra-dimensional limit [51].

At first, it seems implausible that these three theories could arise as different limits of the same theory because they all have different fundamental gauge symmetries. The minimal moose is based on gauging a product group $SU(2) \times SU(2)$, the simple group has the simple group $SU(3)$ gauged, whereas the original holographic Higgs is dual to a CFT with a single copy of $SU(2)$ gauged. How can these theories come from the same M-theory if they have different gauge structures?

The point is that for LHC phenomenology, we only require the low energy degrees of freedom of the three theories to be the same, and indeed, immediately above the electroweak symmetry breaking scale all three theories have only massless $SU(2)$ gauge bosons. The heavy gauge fields will appear at the LHC as new heavy spin-1 modes, and in the spirit of Abbott-Farhi, to first approximation we are free to interpret these heavy modes as either gauge bosons that get a mass via spontaneous symmetry breaking or resonances from some strong dynamics. The little M-theory description will include an $SU(3) \times SU(2)$'s worth of massive gauge bosons, but we can decouple any of the modes that are irrelevant by changing some appropriate gauge couplings.

The toy $SU(3)/SU(2)$ little M-theory can be described by the following moose diagram:

Global : $SU(3)_1$ $SU(3)_m$ $SU(3)_2$

$$\Sigma_1 \qquad\qquad \Sigma_2 \qquad\qquad\qquad (2.12)$$

Gauged : $SU(2)_1$ $SU(3)_m$ $SU(2)_2$

In unitary gauge, an $SU(3) \times SU(2)$'s worth of Goldstone are eaten, yielding $SU(3) \times SU(2)$ massive gauge bosons and massless $SU(2)$ gauge bosons. The link fields are parametrized in terms of the uneaten Goldstones as

$$\Sigma_1 = e^{i\Pi/f_1}, \qquad \Sigma_2 = e^{i\Pi/f_2}. \qquad (2.13)$$

The T-parity limit of this theory is achieved when the gauge couplings g_1 and g_2 and the decay constants f_1 and f_2 are taken to be equal.

It is now straightforward to see how eq. (2.12) can interpolate between the three different theories mentioned above. If we take the g_m gauge coupling to infinity, then we can integrate out the ultra-massive $SU(3)_m$ gauge bosons. If we ignore the mechanism for generating the Higgs quartic, then this yields the correct gauge structure for the minimal moose:

Global : $SU(3)_1$ $SU(3)_2$

$$\Sigma \qquad\qquad\qquad\qquad (2.14)$$

Gauged : $SU(2)_1$ $SU(2)_2$

where

$$\Sigma = \Sigma_1 \Sigma_2. \qquad (2.15)$$

The minimal moose exhibits a collective symmetry breaking structure, in that both g_1 and g_2 must be non-zero for the Higgs boson in Σ to get a radiative potential from gauge loops.

If we take the g_1 and g_2 gauge couplings to infinity, then we can integrate out the ultra-massive $SU(2)_i$ gauge bosons. This will yield the simple group little Higgs. In order to see this, recall from eq. (2.8), that in the little technicolor or hidden local symmetry construction, the moose

$$\text{Global :} \quad \begin{array}{cc} SU(3) & SU(3) \\ \bigcirc \longrightarrow \bigcirc \\ \text{Gauged :} & SU(2)_\rho \end{array} \tag{2.16}$$

turns into a $SU(3)/SU(2)$ nonlinear sigma model when the $SU(2)_\rho$ gauge boson is integrated out. Therefore, when the $SU(2)_i$ gauge bosons are integrated out, we get a theory without an obvious moose description:

$$(SU(3)/SU(2))^2 \text{ non-linear } \sigma\text{-model with } SU(3)_V \text{ gauged} \tag{2.17}$$

which is indeed the simple group theory. Unlike the minimal moose, this theory does not exhibit ordinary collective symmetry breaking. However, the Higgs potential is not quadratically divergent because both f_1 and f_2 must be nonzero for the Higgs boson not to be eaten.

Finally, eq. (2.12) can turn into the original holographic Higgs if we take $f_1 > f_2$. To see this, note that eq. (2.12) can be thought of as the three-site deconstruction of a warped extra dimension with bulk gauge fields and appropriate boundary conditions:

$$\begin{array}{ccc} SU(2) & SU(3) & SU(2) \\ & \text{Bulk} & \\ | & \diagdown\underline{} & | \\ \text{UV Brane} & & \text{IR Brane} \end{array} \tag{2.18}$$

The warp factor is reflected in the different pion decay constants on the links, so there is no natural T-parity limit in this case. The original holographic Higgs exhibits AdS/CFT collective breaking, in the sense that both the IR brane and UV brane boundary conditions must violate the bulk $SU(3)$ symmetry in order for the Higgs to get a radiative potential [52]. To better reproduce an extra dimension, we can add additional $SU(3)$ sites to the middle of the moose.

From a high energy perspective, these three theories have very different philosophies, with different "natural" values for the gauge couplings and the decay constants. For the purposes of LHC phenomenology, however, these theories are just models with novel spin-1 and spin-0 spectra, and the M-theory description is a convenient way to summarize their main features. In the next section, we describe a complete model with hypercharge, custodial $SU(2)$, and three families of standard model fermions.

3. The $\mathrm{Sp}(4)/\mathrm{SO}(4)$ little M-theory

While the toy $\mathrm{SU}(3)/\mathrm{SU}(2)$ moose is good for illustrative purposes, phenomenologically it is not the ideal model. The nonlinear sigma model does not have a custodial $\mathrm{SU}(2)$ symmetry which can lead to large corrections to the ρ parameter. In addition, the up- and down-type quark Yukawa couplings need to be implemented separately and the down-type quark sector is not very appealing in the $\mathrm{SU}(3)/\mathrm{SU}(2)$ coset moose. In principle, it is also possible to write down a moose model which can take a limit of the littlest Higgs. However, most of the phenomenological studies of the little Higgs theories so far have been focused on the littlest Higgs (see, e.g. [53]) It is therefore more useful to provide a model which allows us to study collider phenomenologies for many alternative theories in a uniform way.

The choice for this M-theory is not unique. One can always enlarge the group structure and add more states to the theory so that it can simulate more models and mimic them more accurately. However, it also makes the theory unnecessarily complicated as one needs to break more symmetries and to decouple many extra spurious states in considering various limits. Therefore we will make a compromise between complexity and versatility. We will choose a model which is simple enough to describe and to be implemented in simulation tools and yet can still capture a lot of interesting features of various models. One can always extend the model to incorporate additional features when necessary.

We have made our choice based on the following goals. (1) We want a theoretically consistent and calculable theory. That is, the theory should stay perturbative at energies below the cutoff and there should be no gauge anomalies. (2) There should be no one-loop quadratic sensitivity of the Higgs mass from the top Yukawa coupling and the gauge couplings (except hypercharge), as this is main motivation for much composite Higgs model building. (3) The model should have a custodial $\mathrm{SU}(2)$ symmetry to protect ρ-parameter, and the Yukawa structure should allow for minimal isospin breaking. (4) Minimal flavor violation should be implemented to avoid large flavor-violating effects. (5) To mimic models with extra dimensions we want "KK-partners" of all standard model fermions, but with a dial that can decouple unnecessary particles. (6) The model should have a nice T-symmetric limit because models with or without T-parity have very different phenomenologies. (7) The model should have a lot of dials which allow enough flexibitility to cover a variety of phenomenolgy.

3.1 Gauge/Higgs sector

We will choose the minimal symmetry structure with custodial $\mathrm{SU}(2)$ that allows for both a simple group and a minimal moose limit. We need a group G which contains an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R = \mathrm{SO}(4)$ subgroup and for which the coset space $G/\mathrm{SO}(4)$ contains a Higgs doublet. The minimal choice is $G = \mathrm{SO}(5) \simeq \mathrm{Sp}(4)$. Note that $\mathrm{SO}(5)/\mathrm{SO}(4)$ contains a **4** of $\mathrm{SO}(4)$, which looks like Higgs doublet with custodial symmetry. In our discussion we will use the language of $\mathrm{Sp}(4)$ as the $\mathrm{SU}(2)$ embedding is easier. Appendix A contains a summary of the $\mathrm{Sp}(4)$ group for readers who are unfamiliar with the $\mathrm{Sp}(2N)$ groups. As in the previous section, a three-site moose is chosen so that there is a nice geometric T-symmetric limit.

The master $\mathrm{Sp}(4)/\mathrm{SO}(4)$ moose is as follows:

Global : $\mathrm{Sp}(4)_1$ $\mathrm{Sp}(4)_m$ $\mathrm{Sp}(4)_2$

Gauged : $\mathrm{SU}(2)_{L1} \times \mathrm{U}(1)_R$ $\mathrm{Sp}(4)_m$ $\mathrm{SU}(2)_{L2} \times \mathrm{U}(1)_R$

$$\text{(3.1)}$$

Here we have chosen to gauge only one U(1) with its generator given by

$$T_R = T_{3R1} + T_{3R2} + \frac{1}{2}(B - L). \tag{3.2}$$

After the Σ fields take their vevs, the hypercharge generator is given by

$$T_Y = T_R + T_{3Rm}. \tag{3.3}$$

There are several reasons for this choice of U(1) charges, though it is theoretically consistent to gauge two separate U(1)'s with generators given by

$$T_{R1} = T_{3R1} + \frac{1}{4}(B - L), \qquad T_{R2} = T_{3R2} + \frac{1}{4}(B - L). \tag{3.4}$$

First of all, the quadratically divergent contribution induced by the U(1) gauge interaction is not dangerous for cutoff $\sim 10\,\mathrm{TeV}$ due to the smallness of the gauge coupling. In fact, studies of the precision electroweak constraints on generic little Higgs theories [54–56] show that the massive U(1) gauge boson A_H often causes the biggest problem so it is preferable to just gauge one U(1) from that point of view. In the T-symmetric limit, on the other hand, even though there is no problem with electroweak constraints, it becomes very cumbersome to implement flavor with the extra U(1). A consequence is that the dark matter candidate A_H in T-symmetric models is now replaced by the Goldstone mode that would have been eaten if we had gauged two U(1)s. This may affect the relic density calculation and the detection of the dark matter particles. It would make little difference for the collider phenomenology if it is difficult to tell the spin of the missing particles [57], assuming that either A_H or the corresponding would-be eaten Goldstone boson is the lightest T-odd particle.

The gauge bosons can be classified according to the generators they correspond to. There are $\mathrm{SU}(2)_L$ gauge bosons on each site, $W_{1L}^{\pm,3}$, $W_{2L}^{\pm,3}$, $W_{mL}^{\pm,3}$. For $\mathrm{SU}(2)_R$ gauge bosons there is a complete set, $W_{mR}^{\pm,3}$ in the middle site, but only one additional W_R^3 corresponding to $\mathrm{U}(1)_R$. In addition, there are four more gauge bosons $X_m^{0,1,2,3}$ of the middle site corresponding to the off-diagonal generators of $\mathrm{Sp}(4)$ ($T_{X(0,1,2,3)}$ as given in appendix A). Their masses and mixings are shown in appendix B. Note that in this setup, there are no heavy $\mathrm{SU}(3)_C$ gauge bosons.

Different models are reached by taking limits similar to the ones described in the previous section. Taking $g_m \to \infty$, we can integrate out the middle site and obtain the minimal moose model based on $\mathrm{Sp}(4)$. T-parity corresponds to taking $g_1 = g_2$ and equal decay constants for Σ_1 and Σ_2. In fact, this moose is morally the same as the minimal

moose model with T-parity [31] except that it has fewer links. We do not try to address the Higgs quartic potential through the little Higgs mechanism here because it is quite model dependent and it is not likely to be testable at the LHC other than finding a few more scalar states. On the other hand, if we take the gauge couplings g_1, g_2 of the $SU(2)_{1L}$ and $SU(2)_{2L}$ to infinity, we can integrate out the $SU(2)_{1L,2L}$ gauge bosons and obtain a simple group little Higgs based on the coset space $Sp(4)/SU(2)$ with extra $U(1)$s. These limits are discussed further in section 4.

The Higgs field is contained in

$$\Sigma = \Sigma_1 \Sigma_2, \tag{3.5}$$

with $\langle \Sigma \rangle = 1$. It is convenient to choose a generator basis such that the $Sp(4)$ generators T satisfy

$$TA + AT^T = 0, \qquad A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{3.6}$$

(See appendix A). The Σ vevs break the gauge symmetry $Sp(4) \times SU(2)^2 \times U(1)_R$ down to the standard model $SU(2)_L \times U(1)_Y$. After the Goldstones are eaten, we are left with

$$\Sigma = e^{i\Pi/f_{\text{eff}}}, \qquad \Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & & \\ 0 & 0 & & H \\ & & \phi^0/\sqrt{2} & \phi^- \\ H^\dagger & & \phi^+ & -\phi^0/\sqrt{2} \end{pmatrix} \tag{3.7}$$

where H has the same definition as in section 2.1

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} h^0 & h^- \\ -h^+ & h^{0*} \end{pmatrix}, \tag{3.8}$$

and

$$\frac{1}{f_{\text{eff}}^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2}. \tag{3.9}$$

We see that there are three extra Goldstone bosons in addition to the Higgs. They would have been eaten if we had chosen to gauge a whole $SU(2)_R$ on one of the boundary sites. Indeed, this is precisely what happens in the minimal holographic Higgs model [10]. However, this choice for the gauge structure would prevent us from taking the T-symmetric limit.[4]

The mechanisms for generating the Higgs potential in little Higgs theories are quite model-dependent and there can be one or two (or even more) light Higgs doublets. In the simplest little Higgs model [18], the Higgs potential is generated through the top loop and requires some mild fine-tuning. The minimal moose model contains many link fields and the Higgs potential is given by a collection of complicated plaquette operators. Given this

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[4]It is in principle plausible to add a switch to change between these cases when implementing the model into a collider simulation program. Note that if one were to make this choice, then the fermion sector described in the next subsection would have to be modified.

model-dependence and the fact that the LHC is unlikely to test the little Higgs structure of the Higgs self-couplings, we will simply write down the necessary Higgs potential for the electroweak symmetry breaking without specifying its origin. We show how to do this in a theoretically consistent way in appendix C. This is probably good enough if there is only one light Higgs within the reach of the LHC. It is possible to extend the symmetry structure or number of link fields to allow for multiple Higgses if necessary.

3.2 Fermion sector

We will introduce fermions on each site and the standard model fermions will come from various linear combinations. By changing the linking masses we can change the profiles of the "zero mode" fermions which then looks like localization in extra dimensions. This freedom also allows us to localize the standard model fermions away from the site(s) where the gauge coupling is taken to be large in various limits. On the other hand, it also implies that there will be excited ("KK") fermions. To avoid large flavor-changing effects, we would like to implement minimal flavor violation which either means that the heavy fermions should share the Yukawa structure of the standard model fermions or that they are degenerate among generations. Choosing to consider the T-parity limit forces a small amount of model building upon us, and some additional fields will be needed so that we can independently control the masses of the T-odd fermions.

If we were only interested in third generation quarks, then the simplest fermion sector to get the top and bottom Yukawa couplings would look like

Gauged : $\quad$ SU(2)$_{L1}$ × U(1)$_R$ $\qquad\qquad$ Sp(4)$_m$ $\qquad\qquad$ SU(2)$_{L2}$ × U(1)$_R$

Quarks : $\qquad\qquad$ Q_1^c $\qquad\qquad\qquad\quad$ Q_m $\qquad\qquad\qquad$ Q_2^c
$$\tag{3.10}$$

where

$$Q_i^c = \begin{pmatrix} 0 \\ t_i^c \\ b_i^c \end{pmatrix}, \qquad Q_m = \begin{pmatrix} q_m \\ t_m \\ b_m \end{pmatrix}. \tag{3.11}$$

From the only interactions local in theory space

$$-\mathcal{L}_{\text{Yukawa}} = f_1 Q_m^T \Sigma_1^\dagger \beta_1 Q_1^c + f_2 Q_m^T \Sigma_2 \beta_2 Q_2^c + \text{h.c.}, \tag{3.12}$$

we would generate top and bottom Yukawa couplings and masses for heavy top and bottom partners. The splitting between the top and the bottom comes from the fact that β_i, $i = 1, 2$, are actually custodial SU(2)-violating matrices, $\beta_i = \text{diag}(\beta_{qi}, \beta_{qi}, \beta_{ti}, \beta_{bi})$.

There are two reasons why we want to expand this minimal setup. First, eq. (3.10) only has SU(2)$_L$ singlet fermion partners. More generally, we expect there to be regions of model space with SU(2)$_L$ doublet fermion partners, so we would like to augment eq. (3.10) with additional heavy doublet states that may or may not be decoupled. Note that because of the gauge symmetries and our demand for theory space locality, we cannot have just

SU(2)$_L$ doublet fermion partners; if we were to choose $Q_i^c = (q_i^c \; 0)$ (and reverse the SU(3)$_C$ charges of all the fields), there would be no way to split the top and bottom Yukawa couplings using the interaction in eq. (3.12).

Second, we want to treat all three fermion generations symmetrically in order to implement minimal flavor violation. However, in the T-symmetric limit, eq. (3.12) tells us that there is a fixed ratio between the SM fermion and their T-odd partners masses, i.e. $m_{q'} \sim (m_q/v_{\text{EW}})f_{\text{eff}}$. Therefore, for any reasonable ratio of v_{EW} to f_{eff}, the T-odd partner of, say, the electron would be much lighter than the Z and therefore excluded by the LEP bounds on the Z width. In order to treat the three generations symmetrically, we need a mechanism such that the T-odd partners will all be at the TeV scale, but there will still be a hierarchy among the T-even standard model fields. In order to do so, we will assume that the β_i's are nearly degenerate for all three generations, then will we use a see-saw mechanism to decrease the effective Yukawa coupling for the lighter fermions. While this may see like excessive model building for a phenomenological model, we emphasize that our goal is to have a description of non-SUSY LHC physics that is aware of the various model building challenges that the little hierarchy problem presents.

Visually, for each generation, the complete fermion sector is

Gauged :   SU(2)$_{L1}$ × U(1)$_R$ Sp(4)$_m$ SU(2)$_{L2}$ × U(1)$_R$

Quarks : Q_1, Q_1^c Q_m Q_2, Q_2^c

Leptons : L_1, L_1^c L_m L_2, L_2^c

(3.13)

with floating fermions Q', Q'^c, L', L'^c to enable the flavor see-saw mechanism.[5] Using the third generation as an example, the fermions are embedded as

$$Q_i = \begin{pmatrix} q_i \\ 0 \\ 0 \end{pmatrix}, \quad Q_i^c = \begin{pmatrix} q_i^c \\ t_i^c \\ b_i^c \end{pmatrix}, \quad Q_m = \begin{pmatrix} q_m \\ t_m \\ b_m \end{pmatrix}, \quad Q' = \begin{pmatrix} 0 \\ t' \\ b' \end{pmatrix}, \quad Q'^c = \begin{pmatrix} 0 \\ t'^c \\ b'^c \end{pmatrix}, \quad (3.14)$$

$$L_i = \begin{pmatrix} \ell_i \\ 0 \\ 0 \end{pmatrix}, \quad L_i^c = \begin{pmatrix} \ell_i^c \\ \nu_i^c \\ \tau_i^c \end{pmatrix}, \quad L_m = \begin{pmatrix} \ell_m \\ \nu_m \\ \tau_m \end{pmatrix}, \quad L' = \begin{pmatrix} 0 \\ \nu' \\ \tau' \end{pmatrix}, \quad L'^c = \begin{pmatrix} 0 \\ \nu'^c \\ \tau'^c \end{pmatrix}. \quad (3.15)$$

The anomaly-free fermion charges are given in table 1. Note that Q', Q'^c, L', L'^c contain only the lower two components which are only charged under U(1)$_R$, so they do not need to be associated with any site.

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[5]The floating fermions violate theory space locality, and along with the non-local definition of U(1)$_R$, prevent us from taking a strict holographic composite Higgs limit. On the other hand, without the floating fermions, we know of no way to implement minimal flavor violation in the T-symmetric limit. This tension between T-parity, flavor structure, and theory space locality has been observed before [52], and is probably a generic issue for composite Higgs theories.

		$(\overline{q_i})$	$(t'^c,\overline{t'})$	$(b'^c,\overline{b'})$			$(\overline{\ell_i})$	$(\nu'^c,\overline{\nu'})$	$(\tau'^c,\overline{\tau'})$
	q_m	t_m	b_m	q_i^c	t_i^c	b_i^c	ℓ_m	ν_m	τ_m	ℓ_i^c	ν_i^c	τ_i^c
$\mathrm{SU}(3)_C$	**3**	**3**	**3**	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	–	–	–	–	–	–
$\mathrm{SU}(2)_{Lm}$	**2**	–	–	–	–	–	**2**	–	–	–	–	–
$\mathrm{SU}(2)_{Li}$	–	–	–	**2**	–	–	–	–	–	**2**	–	–
$\mathrm{U}(1)_{3Rm}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
$\mathrm{U}(1)_R$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	1

Table 1: The anomaly-free fermion charges for the $\mathrm{Sp}(4)/\mathrm{SO}(4)$ moose. Note that we have decomposed $\mathrm{Sp}(4)_m$ as $\mathrm{SU}(2)_{Lm} \times \mathrm{U}(1)_{3Rm}$.

We will now write down all couplings local in theory space as well as the leading non-local interaction. Each of these couplings preserves enough of an $\mathrm{Sp}(4)$ global symmetry to avoid one-loop quadratically divergent contributions to the Higgs potential. The reason for including the leading non-local interaction is that in the minimal moose-like limit, the center site is strongly coupled so we want standard model fields to live on the outside sites in order to avoid large four-fermion operators. The non-local interaction will then provide the dominant Yukawa couplings in this limit (See section 4 for a more detailed discussion).

The Yukawa interactions and masses of the fermions are given by

$$-\mathcal{L}_{\mathrm{Yukawa}} = f_1 Q_m^T \Sigma_1^\dagger \beta_1 Q_1^c + f_2 Q_m^T \Sigma_2 \beta_2 Q_2^c + \text{lepton terms} + \text{h.c.}, \tag{3.16}$$

$$-\mathcal{L}_{\mathrm{Mass}} = m_1 Q_1^T Q_1^c + m_2 Q_2^T Q_2^c + \text{lepton terms} + \text{h.c.}. \tag{3.17}$$

$$-\mathcal{L}_{\mathrm{Non-Local}} = \alpha_1 f_{\mathrm{eff}} Q_1^T \Sigma_1 \Sigma_2 Q_2^c + \alpha_2 f_{\mathrm{eff}} Q_2^T \Sigma_2^\dagger \Sigma_1^\dagger Q_1^c + \text{lepton terms} + \text{h.c.}, \tag{3.18}$$

where β_i's are coupling matrices mentioned before. To get small Yukawa couplings to produce the observed fermion mass hierarchies, we can use a see-saw mechanism with the help of Q', Q'^c

$$-\mathcal{L}_{\mathrm{See-Saw}} = Q'^T(F_1 Q_1^c - F_2 Q_2^c) + Q'^T K Q'^c + \text{lepton terms} + \text{h.c.}, \tag{3.19}$$

where $F_i = \mathrm{diag}(0,0,F_{Ti},F_{Bi})$ and $K = \mathrm{diag}(0,0,K_T,K_B)$. In the limit that $F_{Ti,Bi} \gg \beta v_{\mathrm{EW}}, K_{T,B}$, the effective Yukawa couplings for the standard model fermions are suppressed by

$$\lambda_{\mathrm{eff}} \propto \frac{K}{F}. \tag{3.20}$$

For three generations of fermions, F_{Ti}, F_{Bi}, K_T, K_B are 3×3 matrices which encode the flavor structure. As we discuss in the next section, minimal flavor violation corresponds to allowing all of the Yukawa structure to appear only in the K matrix.

4. Exploring the parameter space of little M-theory

Just like in the MSSM, many parameters in little-M theory are already constrained by experimental data. In this section, we will give a general discussion of the constraints one

should be aware of when exploring the little M-theory parameter space. This will also give us an opportunity to show how to approach various "natural", or more familiar, limits.

Unfortunately, because the $U(1)_R$ gauge coupling violates theory space locality, strictly speaking we cannot take a holographic composite Higgs limit as we did in section 2.2. Then again, if we used the U(1) charge assignments of eq. (3.4), then the model would have an extra-dimensional limit as long as the floating Q' and L' fermions were decoupled (i.e. if the K parameter were taken to infinity). Notice that the holographic limit also does not simultaneously allow for T-parity. It would be interesting to explore the phenomenology along that direction, but we will not explicitly discuss this limit further here.

Ignoring the Goldstone sector, the parameter space of little M-theory is defined by

$$
\begin{aligned}
\text{Decay Constants:} \quad & f_1, \quad f_2, \\
\text{Gauge Couplings:} \quad & g_1, \quad g_m, \quad g_2, \quad g_R, \\
\text{Fermion Parameters:} \quad & \beta_i, \quad m_i, \quad \alpha_i, \quad F_i, \quad K.
\end{aligned}
\tag{4.1}
$$

Roughly speaking, the parameter space is spanned by two orthogonal directions: 1) whether T-parity is a good symmetry or not, and 2) whether the model is more simple-group-like or more product-group like. At first glance, both of these directions seem to mainly affect the values of the gauge parameters. Indeed, the T-parity axis corresponds to splitting g_1 and g_2, and the gauge structure axis corresponds to either taking g_m or $g_{1,2}$ large.

As we will see, however, the fermion parameters should be adjusted in concert with the gauge parameters in order to satisfy various experimental bounds. Four-fermion operators generated by integrating out heavy (T-even in the T-parity limit) gauge bosons generically present the strongest constraints on the little M-theory parameter space. In general, one should choose the fermion parameters in such a way that the standard model fermions and gauge bosons have approximately the same profiles in theory space, so that the overlaps between the standard model fermions and the heavy gauge bosons are small.

Regardless of the values of the gauge parameters, flavor- and isospin-violating effects will always impose constraints on the fermion parameters, so we will discuss those bounds first. If we choose to impose minimal flavor and isospin violation, the dominant constraints on the values of the little M-theory parameters come from four-fermion operators involving standard model fields and electroweak precision tests. We will discuss those bounds by first exploring the T-symmetric limit and then seeing how the model building constraints change at the simple group and minimal moose limits on the gauge structure axis.

4.1 Minimal flavor and isospin violation

Because the heavy "KK partners" of the standard model fermions obtain their masses from F_i, $\beta_i f_i$, m_i, and $\alpha_i f_{\text{eff}}$, flavor-changing effects will be induced if these parameters are not flavor universal. To avoid large flavor-changing effects from the heavy fermions it is simplest to implement minimal flavor violation. That is, we can take all the above parameters to be flavor universal and put all the flavor structure in the K matrices, such that $K_{T,B}$ are proportional to the observed Yukawa matrices of the standard model fermions. If we ignore inter-generational mixings, the fermion mass eigenstates for each species are obtained by

diagonalizing a 5×5 mass matrix and there are 2 "KK" modes for each handedness. Some of them may be decoupled in various limits. The mass matrices for the fermions are discussed in appendix B.

The various terms in the fermion Lagrangian also induce mixings between $\mathrm{SU}(2)_L$ doublet fermions and singlet fermions, which can result in isospin-violating shifts of the standard model fermion couplings to the W and Z bosons of the order $v^2/(f^2, F^2, m^2)$. LEP and SLC experiments have tested these couplings of the light fermions to a precision level of 10^{-3} [58]. The simplest solution to this constraint is to have these parameters (except K) respect the isospin symmetry, such that $\beta_{ti} = \beta_{bi}$, $F_{Ti} = F_{Bi}$, and so on for (at least) the light generation quarks and leptons. It is possible to deviate from the assumption of minimal flavor and isospin violation in various corners of the parameter space (especially for the third generation), but one then needs to check various experimental constraints such as $Z \to b\bar{b}$ case by case.

4.2 The T-symmetric limit

The motivation for considering a T-symmetric limit is that T-odd particles cannot lead to tree-level modifications of precision electroweak measurements or four-fermion operators. In the T-symmetric limit, the number of free parameters are greatly reduced. We have $g_1 = g_2 \equiv g$, $f_1 = f_2 \equiv f = \sqrt{2}f_{\mathrm{eff}}$, $m_1 = m_2 \equiv m$, $\beta_1 = \beta_2 \equiv \beta$, $\alpha_1 = \alpha_2 \equiv \alpha$, $F_1 = F_2 \equiv F$. The mass eigenstates divide into T-even and T-odd states. T-parity is defined as the geometric symmetry of the moose diagram by exchanging site-1 and site-2, with a twist by $\Omega = \mathrm{diag}(1, 1, -1, -1)$. The Ω twist flips the parity of the bottom two components of the fermions and the off-diagonal 2×2 blocks of the gauge fields and the Goldstone fields, so that all standard model fields (including the Higgs) are even under T-parity.

The even and odd states in the gauge sector are

$$
\begin{aligned}
&T\text{-even:} \quad W_{+L}^{\pm,3} \equiv \frac{1}{\sqrt{2}}(W_{1L}^{\pm,3} + W_{2L}^{\pm,3}), \ \ W_{mL}^{\pm,3}, \ W_R^3, \ W_{mR}^{\pm,3} \\
&T\text{-odd:} \quad W_{-L}^{\pm,3} \equiv \frac{1}{\sqrt{2}}(W_{1L}^{\pm,3} - W_{2L}^{\pm,3}), \ \ X^{\pm} \equiv \frac{1}{\sqrt{2}}(X^2 \mp iX^1), \\
&\qquad\qquad X^{n(*)} \equiv \frac{1}{\sqrt{2}}(X^0 \pm iX^3)
\end{aligned}
\tag{4.2}
$$

and in the fermion sector are

$$
\begin{aligned}
T\text{-even:} \quad & q_m, \ q_+ \equiv \frac{1}{\sqrt{2}}(q_1 + q_2), \ q_+^c \equiv \frac{1}{\sqrt{2}}(q_1^c + q_2^c), \\
& t_+^c(b_+^c) \equiv \frac{1}{\sqrt{2}}\Big(t_1^c(b_1^c) - t_2^c(b_2^c)\Big), \ t'(b'), \ t'^c(b'^c)
\end{aligned}
$$

$$
\begin{aligned}
T\text{-odd:} \quad & t_m(b_m), \ q_- \equiv \frac{1}{\sqrt{2}}(q_1 - q_2), \ q_-^c \equiv \frac{1}{\sqrt{2}}(q_1^c - q_2^c), \\
& t_-^c(b_-^c) \equiv \frac{1}{\sqrt{2}}\Big(t_1^c(b_1^c) + t_2^c(b_2^c)\Big),
\end{aligned}
\tag{4.3}
$$

and in the scalar sector are

$$
\begin{aligned}
T\text{-even:} &\quad H, \\
T\text{-odd:} &\quad \phi^{\pm,0}.
\end{aligned}
\tag{4.4}
$$

States with same quantum numbers under the standard model and same T-parity mix in general. Of course after electroweak symmetry breaking, there are also mixings between states transforming under $\mathrm{SU}(2)_L$ and states transforming under $\mathrm{SU}(2)_R$. The masses of these states are calculated in appendix B.

In the T-symmetric limit with minimal flavor and isospin violation, the dominant constraints on the values of the little M-theory parameters come from the four-fermion operators involving standard model fields and loop corrections to the $Z \to b\bar{b}$ vertex and the ρ parameter. At tree-level, the four fermion operators come from integrating out heavy T-even gauge bosons, and we will discuss these constraints more in the next subsection. The loop corrections only impose mild constraints and the f's can be well below $1\,\mathrm{TeV}$ [32], making the new particles more accessible at the LHC. In fact, the T-symmetric limit is probably the least constrained scenario, and we expect there to exist a large region of "safe" parameter space both at and near the T-parity limit. Of course if we deviate from minimal flavor and isospin violation, there are other constraints to worry about but they are model-dependent.

Deviations from T-parity are generically subject to stronger electroweak precision constraints, which would have to be checked by hand. Without T-parity, one can have tree-level modifications of precision electroweak parameters. Having said that, we expect that for the $\mathrm{Sp}(4)/\mathrm{SO}(4)$ model with a custodial $\mathrm{SU}(2)$ symmetry and only one $\mathrm{U}(1)$ gauged, the constraints from oblique parameters should be weak. We do expect four-fermion operators to be dangerous away from the T-parity limit.

4.3 The gauge structure axis

As we adjust the ratio of g_m to $g_{1,2}$, we interpolate between simple group and product group gauge structures at low energies. However, if we are not careful, then the effect of the "decoupled" heavy gauge bosons can be large. The mass of the heavy gauge bosons scale as $\sim gf$ but their couplings scales like $\sim g$, so at low energies one generically once expects to find non-decoupling four-fermion operators suppressed only by $1/f_{\mathrm{eff}}^2$. In order to soften the effects of these heavy gauge bosons, we should arrange the standard model fermion wavefunctions in theory space to have small overlaps with the strong-coupling site(s).

In the next subsection, we show explicitly how to minimize the size of four-fermion operators at the two extremes of the gauge structure axis, and comment further on precision electroweak constraints away from the T-parity limit. In both the simple group and minimal moose limits, it is also possible to decouple unnecessary modes if one does not wish to impose T-parity.

4.3.1 The simple group limit

As already mentioned, the simple group little Higgs limit is obtained by taking the gauge couplings g_1 and g_2 of the outer sites to infinity. In this case, the $SU(2)_L$ gauge bosons on sites 1 and 2, $W_{1L}^{\pm,3}$ and $W_{2L}^{\pm,3}$, become heavy and decouple from the low energy spectrum. To further decouple their effects in order to avoid possible large four-fermion interactions induced by them [59], the standard model $SU(2)$-doublet fermions should be mostly localized in the middle site. This can be done by taking m_1 and m_2 large. In the limit $m_{1,2} \to \infty$, q_1, q_1^c, q_2, q_2^c decouple and the fermion spectrum simplifies. If one does not need to take the T-symmetric limit, the Yukawa flavor structure can come from one of the β's. In this case the fermion sector can be further simplified by taking K to infinity to decouple $t'(b')$, $t'^c(b'^c)$. The standard model Yukawa couplings can be obtained by taking

$$\beta_{t(b)1} \ll \beta_{t(b)2} \sim \mathcal{O}(1), \tag{4.5}$$

and $t_m(b_m)$ and $t_2^c(b_2^c)$ will acquire a mass of $\beta_{t(b)2} f_2 \sim$ TeV. The $\beta_{t(b)2}$ should be family-universal to avoid large flavor-changing effects. The standard model fermions and Yukawa couplings are approximately given by $q_{SM} \sim q_m$, $t_{SM}^c(b_{SM}^c) \sim t_1^c(b_1^c)$ and $\lambda_{t(b)} \sim \beta_{t(b)1} s/2$, where $s = f_2/\sqrt{f_1^2 + f_2^2}$, up to small corrections.

In the $SU(3)/SU(2)$ simple group little Higgs model, the strongest constraints come from the $Z - Z'$ mixing and the four-fermion interactions induced by Z' [18], where Z' is the gauge boson corresponding to the generator T_8. As a result $\sqrt{f_1^2 + f_2^2}$ is required to be larger than a couple TeV. The fine-tuning can be reduced by taking unequal f_1 and f_2 which allows the freedom to adjust the masses of the top partner and gauge partners relatively. In our model, on the other hand, there is a complete custodial $SU(2)$ multiplet of $W_{mR}^{\pm,3}$ gauge bosons which can mix with the standard model $W_{mL}^{\pm,3}$ gauge bosons after electroweak symmetry breaking. As a result, the mixing does not induce any further custodial $SU(2)$ violation unlike in the $SU(3)/SU(2)$ model. In addition, the standard model fermions are not charged under the $SU(2)_{Rm}$ subgroup of $Sp(4)_m$. The only four-fermion operators induced by heavy gauge bosons at the leading order (not suppressed by v^2/f^2) come from the $U(1)_R$ component of Z_R, which is a combination of W_{mR}^3 and $U(1)_R$ gauge field W_R^3 (see appendix B). However, they are suppressed by the smallness of the component W_R^3 in Z_R and the $U(1)_R$ gauge coupling. The constraint from the electroweak precision measurements can therefore be much weaker.

4.3.2 The minimal moose limit

The minimal moose little Higgs limit is obtained by taking g_m to infinity. In this case we can integrate out the middle site and all gauge bosons of the middle site decouple. To avoid large residual four-fermion interactions induced by them, the standard model fermions should live away from the middle site. This can be achieved by taking $\beta_{q1} f_1$, $\beta_{q2} f_2$ to be larger than $\alpha_2 f_{\rm eff}$, m_1, $\alpha_1 f_{\rm eff}$, m_2. More specifically, if we do not need to take the T-symmetric limit, we can remove Q_m and Q_2^c from the low energy spectrum as follows: take $\beta_{q2} f_2$ large ($\gg m_2, \alpha_1 f_{\rm eff}$) so that q_m and q_2^c decouple, and $\beta_{t(b)2} f_2$ large ($> \beta_{t(b)1} f_1$) so that $t_m(b_m)$ and $t_2^c(b_2^c)$ decouple. We are left with a complete $Sp(4)$ multiplet

$Q_1^c = (q_1^c, t_1^c, b_1^c)^T$ and q_1, q_2, $t'(b')$, $t'^c(b'^c)$. Two pairs of the quark-antiquark get $\sim \text{TeV}$ masses from m_1 and $F_{T(B)1}$. They play the roles of the heavy fermions which cancel the one-loop quadratic divergence from the standard model top quark loop. The Yukawa couplings are $\lambda_{t(b)} \sim \alpha_2 K_{T(B)}/(2F_{T(B)1})$ (assuming $\alpha_2 f_{\text{eff}} \ll m_1$). With minimal flavor and isospin violation, the standard model Yukawa structure and the top-bottom splitting all come from $K_{T,B}$. The above discussion is more transparent by examining the fermion mass matrix given in appendix B.

Because the model has a custodial SU(2) symmetry and we only gauge one U(1), the strongest constraint comes from the remaining four-fermion interactions generated by the W_L' and Z_L' gauge bosons (the heavy combinations of the W_{1L}, W_{2L} and the U(1)$_R$ gauge bosons). They are required to be heavier than a few TeV if the standard model fermions are localized on one site, but the constraint can be greatly relaxed by taking the T-symmetric limit. In particular, the W_L' and Z_L' gauge bosons are T-odd, therefore they cannot contribute at tree-level to any four-fermion or precision electroweak operator.

5. Comments on collider phenomenology

Little M-theory is a framework which captures the dominant features of composite Higgs models. We expect this framework to exhibit a rich phenomenology with many novel features which deserve detailed studies. In particular, a comprehensive study of the inverse map from LHC signature space to little M-theory parameter space could and should be undertaken, and it will be interesting to see whether little M-theory with T-parity exhibits the same degeneracy structure as was found in the MSSM [60]. In this section, we confine ourselves to qualitative comments about the phenomenology of little M-theory. The collider phenomenologies are very different for models with or without T-parity. We start our discussion in the T-symmetric limit because it is preferred by experimental constraints.

The initial signal of a T-symmetric composite Higgs scenario will be significant and dramatic, because the QCD pair production cross-section for the fermionic partners of standard model quarks will be large. The decays of the T-odd quark partners typically proceed through decay chains involving gauge boson partners as well as leptonic partners, terminating in the lightest T-odd particle. Therefore, the typical signature will be jets, leptons and large missing energy. Without measuring at least some details of the spectrum and couplings, it is probably indistinguishable from a supersymmetric scenario. Therefore, it should be included, along with SUSY, in the collection of initial candidates of possible scenarios if such signatures are found.

There will be exotic gauge bosons, like $W_R^\pm$, Z_R (T-even) and X (T-odd). (Z_R is a massive combination of W_{mR}^3 and the U(1)$_R$ gauge field W_R^3 defined in appendix B.) Generically, it is not possible to make all of the heavy gauge bosons on the middle site odd under T-parity. This is an interesting difference between a pure product group structure, which only requires the two outside sites, and a simple group structure, which is represented here by the middle site. Because of this fact, a smoking gun signature for the existence of some simple group structure is the existence of these exotic gauge bosons. In this model, Z_R can easily be seen as a resonance by Drell-Yan production. However, this is not

a distinguishing feature between little-M theory and SUSY, as extensions of the MSSM could certainly have new gauge sectors. $W_R^\pm$ only couples to standard model fermions through mixings after electroweak symmetry breaking. The couplings are suppressed by $\mathcal{O}(v^2/f^2) \sim 10^{-2}$, so they will be difficult to produce directly. X gauge bosons are T-odd so they need to be pair-produced or produced together with another T-odd state. Their decays will have missing energy which makes it a challenge to reconstruct their identities. In the case of $g_1 \sim g_2 \sim g_m$, we will be able to produce both heavier combinations of $SU(2)_L$, W_L', W_L'' (from eq. (B.6)), as well as some of the exotics. It is obviously an interesting new benchmark to explore. In particular, the verification of cancellation of quadratic divergences is expected be more complicated than both the simple and product group limits.

In our construction, the lightest neutral T-odd particle is expected to be the ϕ^0 scalar. (A neutral heavy gauge boson might be the lightest neutral mode in extreme regions of parameter space.) However, it could be challenging to measure its spin especially if it is stable. (See, however, ref. [61].) The mass of ϕ^0 and its couplings to Higgs are essentially free parameters in this theory as discussed in appendix C. As a result, it could have interesting consequences for Higgs physics. For example, the Higgs could have a large invisible branching ratio to such a scalar via a $h^\dagger h \phi^\dagger \phi$ coupling.

Next we consider the collider phenomenology away from the T-parity limit. Notice that T-parity violation is probably only constrained by precision electroweak measurements and flavor physics, and is therefore not nearly as dangerous as lepton or baryon number violating R-parity breaking in SUSY. Without T-parity, the new particles do not always need to be pair-produced and there is no new stable neutral particle to give the missing energy signature. They can be searched for by looking for peaks in invariant mass distributions. For new gauge bosons, Z_R and W_L' in general have unsuppressed couplings to standard model fermions and can be produced easily, dominated by the Drell-Yan process. The couplings of the X gauge bosons to standard model fermions are suppressed by v/f, but LHC can still have a significant reach ($\sim 2\,\text{TeV}$) for them [62]. On the other hand, discovering $W_R^\pm$ through direct production in this model will be challenging as their couplings to standard model fermions are suppressed by v^2/f^2. It may be more promising if they appear as decay products of other new particles. Quark partners will have large QCD production cross sections if they are not too heavy. In general they can decay to a standard model quark and a new heavy gauge boson, which then subsequently decay to standard model particles. If the T-parity violations are small, the ratio of single and pair productions may provide a measurement of the size of such violations. The decays of the approximate T-odd particles will mostly follow the decay chains of the T-symmetric model until the last step, where the lightest approximate T-odd particle decay into standard model particles.

With the accumulation of higher luminosity, one can ask more detailed questions. For example, what is the underlying global symmetry structure that protects the Higgs mass? The moose presented here is the minimal one that preserves custodial SU(2) and in this sense is a good starting point for reconstructing the composite symmetry structure. On the other hand, if we get more detailed information about the exotics, we should be able to determine the global symmetry structure precisely. For example, if there are fewer exotic gauge bosons, such as no $W_R^\pm$, we might want to consider the global symmetry structure

considered in section 2.2. Alternatively, to account for more scalar states, we could increase the size of the symmetry breaking G/H coset space.

For the same reason, we will need to adjust the fermion structure based on what we observe, in particular after we have some idea of whether left- and/or right-handed partners of the standard model fermions have been produced. In the T-symmetric limit, it will be important to study whether or not one can actually tell the difference between left- and right-handed partner production at the LHC, and just like in trying to distinguish between left- or right-handed squarks, lepton production from cascade decays may be crucial in determining whether or not the new partners have $SU(2)_L$ charges [60]. If T-even partner fermions are accessible at the LHC, it may be easier to determine their charges, spins, and couplings as their decays do not necessarily yield large missing energy. Similar to the second KK resonances in UED, detection of even fermionic states, as resonances, could be also used as a clue which distinguishes this scenario from R-parity conserving low energy supersymmetry.

Another interesting feature to pay attention to is the existence of one or more gluon partners. While composite models do not require any heavy color octets, they are generically present in models with extra-dimensional or holographic interpretations. Though the production cross section for gluon partners can be large, it may be difficult to distinguish from partner fermion production without some information about jet charges or lepton charge (a)symmetries. This is particularly true for the T-parity conserving case. If we observe gluon partners, it would then be necessary to add additional structure to the moose, though if we ignore anomaly cancellation, a KK-gluon can be accommodated in little M-theory by introducing separate $SU(3)_C$ groups on each site with additional link fields to break $SU(3)^3$ down to the diagonal.

Notice that in order to go to the limits described in section 4, we need to take certain gauge couplings to be strong. This means that in the parameter space of the little-M theory, there are regions where certain gauge modes are quite strongly coupled and yet not completely decoupled. Such regions could be challenging to simulate accurately with tree-level Monte Carlo tools, and one should be careful making statements about the discovery reach for these strongly coupled modes. Note that we do not expect these strongly coupled gauge bosons to form bound states with fermions, since the mass of the gauge bosons also scale up with the coupling.

Finally, away from the T-symmetric limit, little M-theory will mimic a lot of the phenomenology of Higgsless theories as well. Generically, one expects to see heavy gauge bosons at the LHC before one sees the Higgs, and there may initially be some confusion about whether the new spin-1 modes have the right couplings to unitarize W-W scattering. If there is evidence for both new scalar and vector particles with $SU(2)_L$ couplings, then we would have to figure out a way to distinguish between a composite Higgs model with vector partners and Higgsless theory with extra technipions.

6. Outlook

In this paper, we have used the fact that different ultraviolet theories can yield the same low energy physics to develop a general framework for describing non-SUSY physics at LHC

energy scales. The $Sp(4)/SO(4)$ little M-theory interpolates between simple group-like and minimal moose-like composite Higgs models, allowing for rich collider phenomenology. While the $Sp(4)/SO(4)$ moose is by no means the unique choice for describing non-SUSY physics, it is a well-motivated model that has the minimal symmetry structure compatible with custodial $SU(2)$.

Of course, at higher energies, different fundamental theories can be distinguished from their M-theory approximations. If a tower of KK modes is seen whose masses fall at the roots of Bessel functions, then a warped extra dimension (or a strongly coupled CFT) would be the most straightforward explanation and a moose description would be needlessly cumbersome. Similarly, if KK gravitons are seen at the LHC, then a moose description would be inappropriate, though there has been progress in developing healthy lattice descriptions of gravitational warped dimensions [63, 64]. However, if we take the little hierarchy problem seriously, then we do not expect to see strong dynamics or a plethora of KK states at the TeV scale. Rather, we expect to find weakly coupled new physics, and theory space is an especially convenient framework for describing new weakly coupled non-SUSY physics.

Though we have focused on composite Higgs models in this paper, with suitable modifications it is also possible to construct M-theories that interpolate between composite Higgs and UED theories. Ignoring the Higgs sector, a little M-theory with T-parity could describe the phenomenology of lower lying KK-modes in UED with KK-parity. There are two important differences one would have to address to make this possible. First, in UED there are same-statistics partners for every standard model field whereas in composite Higgs theories there need not be the analog of the KK gluon. Additional ingredients need to be added to little M-theory in order to describe those states. Second, the KK fermions are Dirac in UED, so additional sites and fermions need to be added to incorporate this fact. Although equivalent in principle, either an extra-dimensional or a moose description could be more useful depending on the spectrum discovered at the LHC. If evenly spaced resonances nearly degenerate in mass are discovered, UED will be undoubtedly be a much simpler framework to work with. Otherwise, little M-theory type moose models would be more useful since they allow for more general mass relations.

It is also interesting that the variety of moose models include Higgsless models and the low-lying resonances of technicolor as well [65 – 72]. In both composite Higgs and Higgsless theories, the longitudinal modes of the W and Z bosons can be thought of as living in the A_5 component of a bulk gauge field. Similarly, in the vector limit [73], the ρ meson and other light resonances of scaled-up QCD can be described by multi-site mooses. However, there are differences in detailed realizations between composite Higgs and Higgsless theories that make such an interpolation less useful. For example, in Higgsless theories W-W scattering is unitarized by a tower of spin-1 modes instead of a spin-0 physical Higgs mode, so a master M-theory would require both sets of unitarizing fields. In addition, there is no useful notion of T-parity in Higgsless theories, because W-W scattering has to be unitarized by tree-level exchange of spin-1 modes, so a T-symmetric Higgsless theory would have additional T-odd states without improving constraints from precision electroweak measurements. Therefore, it is probably natural to think of composite Higgs and Higgsless theories as two classes of

moose models, just as the MSSM and the NMSSM are two classes of SUSY models with different approaches to the Higgs sector.

Besides offering an interesting model for collider studies, little M-theory also suggests an interesting philosophy for physics in the LHC era. Because there is no simple ultraviolet completion of the $\mathrm{Sp}(4)/\mathrm{SO}(4)$ moose, little M-theory is unlikely to satisfy top-down physicists who pine for UV complete models. Because the $\mathrm{Sp}(4)/\mathrm{SO}(4)$ moose contains fixed relationships among some of the parameters, little M-theory is unlikely to satisfy bottom-up physicists who would rather measure Lagrangian parameters with no theoretical biases. However, if there is a natural solution to the hierarchy problem and a compelling explanation for the little hierarchy, then it is likely that both top-down and bottom-up approaches will be necessary to decipher LHC physics. This is especially true if there is T-parity and much of the decay topology information is lost as missing energy at the LHC. As a theoretically consistent model with a low $10\,\mathrm{TeV}$ cutoff, little M-theory suggests a compromise between the top-down and bottom-up approaches particularly well-suited for the LHC.

Acknowledgments

A. $\mathrm{Sp}(4)$ representations

The elements of $\mathrm{Sp}(4)$ consist of unitary matrices P that satisfy

$$PAP^T = A, \tag{A.1}$$

where A is an anti-symmetric matrix. In terms of generators T, every element of $\mathrm{Sp}(4)$ can be written as $P = e^{iT\phi}$, where

$$TA + AT^T = 0, \qquad \mathrm{tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab}. \tag{A.2}$$

For convenience, we work in a basis where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{A.3}$$

and the 10 $\mathrm{Sp}(4)$ generators that satisfy these conditions are

$$T_{Li} = \frac{1}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & 0 \end{pmatrix}, \quad T_{Ri} = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ 0 & \sigma^i \end{pmatrix}, \quad T_{X0} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T_{Xi} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & i\sigma^i \\ -i\sigma^i & 0 \end{pmatrix}. \tag{A.4}$$

$T_{L,R}$ generate the $\mathrm{SU}(2)_{L,R}$ subgroups of $\mathrm{Sp}(4)$.

B. Mass spectrum of the $\mathrm{Sp}(4)/\mathrm{SO}(4)$ little M-theory

In this appendix, we discuss the mass spectrum of the $\mathrm{Sp}(4)/\mathrm{SO}(4)$ little M-theory. In the gauge sector, before electroweak symmetry breaking, gauge bosons can be classified according to their transformation properties under $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_{T_{3R}}$. and there are mixings within each class. Only the middle site has $W_R^\pm$ and off-diagonal X gauge bosons and their masses are given by

$$M_{W_{mR}^\pm}^2 = M_{X_m}^2 = g_m^2(f_1^2 + f_2^2). \tag{B.1}$$

W_{mR}^3 and W_R^3 mix through the Σ vevs and the mass matrix is

$$M_{(W_{mR}^3, W_R^3)}^2 = \begin{pmatrix} g_m^2(f_1^2 + f_2^2) & -g_m g_R(f_1^2 + f_2^2) \\ -g_m g_R(f_1^2 + f_2^2) & g_R^2(f_1^2 + f_2^2). \end{pmatrix} \tag{B.2}$$

The massless combination

$$B = \frac{1}{\sqrt{g_R^2 + g_m^2}} \left(g_R W_{mR}^3 + g_m W_R^3 \right) \tag{B.3}$$

is identified with the hypercharge gauge boson of the standard model. The standard model hypercharge coupling g' is given by

$$\frac{1}{g'^2} = \frac{1}{g_R^2} + \frac{1}{g_m^2}. \tag{B.4}$$

The orthogonal combination Z_R acquires a mass-squared of

$$(g_m^2 + g_R^2)(f_1^2 + f_2^2). \tag{B.5}$$

There is a set of $\mathrm{SU}(2)_L$ gauge bosons on each site. Their mass-squareds form a 3×3 matrix,

$$M_{(W_{1L}, W_{mL}, W_{2L})}^2 = \begin{pmatrix} g_1^2 f_1^2 & -g_1 g_m f_1^2 & 0 \\ -g_1 g_m f_1^2 & g_m^2(f_1^2 + f_2^2) & -g_2 g_m f_2^2 \\ 0 & -g_2 g_m f_2^2 & g_2^2 f_2^2 \end{pmatrix}. \tag{B.6}$$

After diagonalizing the matrices, there is one massless combination,

$$W^{\pm,3} = \frac{1}{\sqrt{g_1^{-2} + g_m^{-2} + g_2^{-2}}} \left(\frac{W_{1L}^{\pm,3}}{g_1} + \frac{W_{mL}^{\pm,3}}{g_m} + \frac{W_{2L}^{\pm,3}}{g_2} \right) \tag{B.7}$$

which is identified with the standard model W gauge bosons. The standard model $\mathrm{SU}(2)_L$ gauge coupling g is given by

$$\frac{1}{g^2} = \frac{1}{g_1^2} + \frac{1}{g_m^2} + \frac{1}{g_2^2}. \tag{B.8}$$

There are two massive modes with mass-squareds of

$$\frac{1}{2} \left(g_1^2 f_1^2 + g_2^2 f_2^2 + g_m^2(f_1^2 + f_2^2) \right.$$

$$\pm \sqrt{(g_1^2 f_1^2 + g_2^2 f_2^2 + g_m^2(f_1^2 + f_2^2))^2 - 4(g_1^2 g_m^2 + g_2^2 g_m^2 + g_1^2 g_2^2)f_1^2 f_2^2}\Big). \quad \text{(B.9)}$$

In the T-symmetric limit, $g_1 = g_2 = \bar{g}$, $f_1 = f_2 = f = \sqrt{2}f_{\text{eff}}$, they reduce to

$$M_{W_L,\text{odd}}^2 = 2\bar{g}^2 f_{\text{eff}}^2, \quad M_{W_L,\text{even}}^2 = 2(\bar{g}^2 + 2g_m^2)f_{\text{eff}}^2. \quad \text{(B.10)}$$

After electroweak symmetry breaking, $W^\pm$ and $Z = \cos\theta_W W^3 - \sin\theta_W B$ acquire masses and only the photon $A = \sin\theta_W W^3 + \cos\theta_W B$ is left massless. The heavy gauge bosons (with mass $\sim gf$) also receive corrections from the electroweak symmetry breaking and there are further mixings among states with the same electric charge (and T-parity if it is a good symmetry). The corrections are small though ($\mathcal{O}(v^2/f^2)$).

We now discuss the fermion mass spectrum and use the third generation quarks as an example. Before electroweak symmetry breaking, there are 3 q's and 2 q^c's for the doublets, so one combination of q_1, q_2 and q_m remains massless. Similarly, there are 2 t's and 3 t^c's for the singlets so one combinations of t_1^c, t_2^c and t'^c remains massless. They can be identified as the standard model top-bottom quark doublet and top quark singlet respectively. They acquire a mass only after the electroweak symmetry breaking. The eigenstates and eigenvalues are obtained by diagonalizing a 5×5 mass matrix (ignoring inter-generation mixings). The 5×5 mass matrix to order $\mathcal{O}(v)$ is given by

$$
\begin{array}{ccccc}
q_1^c & q_2^c & t_1^c(b_1^c) & t_2^c(b_2^c) & t'^c(b'^c)
\end{array}
$$

$$
\begin{array}{c}
q_1 \\ q_2 \\ q_m \\ t_m(b_m) \\ t'(b')
\end{array}
\begin{pmatrix}
m_1 & \alpha_1 f_{\text{eff}} & 0 & \frac{i}{2\sqrt{2}}\alpha_1 v & 0 \\
\alpha_2 f_{\text{eff}} & m_2 & -\frac{i}{2\sqrt{2}}\alpha_2 v & 0 & 0 \\
\beta_{q1} f_1 & \beta_{q2} f_2 & -\frac{i}{2\sqrt{2}}s\beta_{t(b)1}v & \frac{i}{2\sqrt{2}}c\beta_{t(b)2}v & 0 \\
-\frac{i}{2\sqrt{2}}s\beta_{q1}v & \frac{i}{2\sqrt{2}}c\beta_{q2}v & \beta_{t(b)1}f_1 & \beta_{t(b)2}f_2 & 0 \\
0 & 0 & F_{T(B)1} & -F_{T(B)2} & K_{T(B)}
\end{pmatrix}, \quad \text{(B.11)}
$$

where

$$c = \frac{f_1}{\sqrt{f_1^2 + f_2^2}}, \quad s = \frac{f_2}{\sqrt{f_1^2 + f_2^2}}, \quad f_{\text{eff}} = sf_1 = cf_2. \quad \text{(B.12)}$$

In the T-symmetric limit, $m_1 = m_2 = m$, $F_1 = F_2 = F$, $\beta_1 = \beta_2 = \beta$, $\alpha_1 = \alpha_2 = \alpha$, $s = c = 1/\sqrt{2}$, $f_1 = f_2 = f = \sqrt{2}f_{\text{eff}}$, the even and odd states decouple. The mass matrix for the even states is

$$
\begin{array}{ccc}
q_+^c & t_+^c(b_+^c) & t'^c(b'^c)
\end{array}
$$

$$
\begin{array}{c}
q_+ \\ q_m \\ t'
\end{array}
\begin{pmatrix}
m + \alpha f_{\text{eff}} & -\frac{i}{2\sqrt{2}}\alpha v & 0 \\
2\beta_q f_{\text{eff}} & -\frac{i}{2\sqrt{2}}\beta_{t(b)}v & 0 \\
0 & \sqrt{2}F_{T(B)} & K_{T(B)}
\end{pmatrix}, \quad \text{(B.13)}
$$

and before electroweak symmetry breaking, the heavy masses are

$$\sqrt{2F_{T(B)}^2 + K_{T(B)}^2}, \quad \sqrt{(m + \alpha f_{\text{eff}})^2 + 4f_{\text{eff}}^2 \beta_q^2}. \quad \text{(B.14)}$$

Apart from $\mathcal{O}(v^2/f^2)$ corrections, the standard model zero modes are

$$q_{\rm SM} = \frac{1}{\sqrt{(m + \alpha f_{\rm eff})^2 + 4f_{\rm eff}^2 \beta_q^2}} \left(2f_{\rm eff}\beta_q q_+ - (m + \alpha f_{\rm eff})q_m\right),$$

$$t_{\rm SM}^c = \frac{1}{\sqrt{K_T^2 + 2F_T^2}} \left(K_T t_+^c - \sqrt{2}F_T t'^c\right), \tag{B.15}$$

and the Yukawa coupling is

$$\lambda_{\rm SM} = \frac{iK_{T(B)}(\beta_{t(b)}(m + \alpha f_{\rm eff}) - 2\beta_q \alpha f_{\rm eff})}{\sqrt{4(K_{T(B)}^2 + 2F_{T(B)}^2)(4\beta_q^2 f_{\rm eff}^2 + (m + \alpha f_{\rm eff})^2)}}. \tag{B.16}$$

In the T-odd sector, the fermion mass matrix is

$$
\begin{array}{cc}
 & q_-^c \qquad\quad t_-^c\,(b_-^c) \\[4pt]
\begin{array}{c} q_- \\ t_m(b_m) \end{array} & \begin{pmatrix} m - \alpha f_{\rm eff} & \frac{i}{2\sqrt{2}}\alpha v \\ -\frac{i}{2\sqrt{2}}\beta_q v & 2\beta_{t(b)} f_{\rm eff} \end{pmatrix}
\end{array} . \tag{B.17}
$$

C. Adjusting Goldstone masses and interactions

In this appendix, we discuss the potential and mass spectrum of the pseudo-Goldstone bosons, including the Higgs field and the extra scalars $\phi^{\pm,0}$. As we mentioned above, how the Higgs potential arises in various little Higgs theories is very model-dependent. So, instead of specifying some particular mechanism to generate the Higgs potential, we will simply parametrize the scalar potential through appropriate operators made from the Σ field. Non-trivial potentials for the Goldstones can be written down with insertions of symmetry-breaking spurions. In the custodial SU(2) limit, the potential for Σ can be written as

$$V(\Sigma) = f^4 \bigg[\kappa_1 \,{\rm tr}\,\Sigma\Theta\Sigma^\dagger\Theta + \kappa_2 \,{\rm tr}\,\Sigma\Theta\Sigma\Theta + \kappa_3 \,{\rm tr}\,\Sigma\Theta\Sigma\Theta\Sigma\Theta + \kappa_4 \,{\rm tr}\,\Sigma\Theta\Sigma\Theta\Sigma^\dagger\Theta \tag{C.1}$$

$$+ \kappa_5 ({\rm tr}\,\Sigma\Theta\Sigma^\dagger\Theta)^2 + \kappa_6 \,{\rm tr}(\Sigma\Theta\Sigma^\dagger\Theta)^2 + \cdots + {\rm h.c.}\bigg] \tag{C.2}$$

where $\Theta = (I - \Omega)/2 = {\rm diag}(0,0,1,1)$ is the spurion matrix for the breaking of Sp(4). Θ can be inserted between two Σ's because there is only one common U(1)$_R$ gauged in the bottom two rows and columns. Each of these terms contains various combinations of masses and interactions of the Goldstones, for instance,

$$f^4 \,{\rm tr}\,\Sigma\,\Theta\,\Sigma^\dagger\,\Theta = f^4 \left(2 - \frac{h^\dagger h}{2f^2} + \cdots\right),$$

$$f^4 \,{\rm tr}\,\Sigma\,\Theta\,\Sigma\,\Theta = f^4 \left(2 - \frac{h^\dagger h}{2f^2} - 2\frac{\phi^+\phi^-}{f^2} - \frac{(\phi^0)^2}{f^2} + \cdots\right). \tag{C.3}$$

Therefore, by choosing the coefficients κ_1, κ_2,..., one can produce any scalar potential consistent with the symmetries. Under the $\mathrm{SU}(2)_L$ (gauged) and $\mathrm{SU}(2)_R$ (custodial, with the $\mathrm{U}(1)_R$ subgroup gauged), the Higgs multiplet

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} h^0 & h^- \\ -h^+ & h^{0*} \end{pmatrix} \tag{C.4}$$

transforms as $(\mathbf{2}, \mathbf{2})$ and the remaining Goldstones

$$\Phi = \begin{pmatrix} \phi^0/2 & \phi^-/\sqrt{2} \\ \phi^+/\sqrt{2} & -\phi^0/2 \end{pmatrix} \tag{C.5}$$

transforms as $(\mathbf{1}, \mathbf{3})$. The leading scalar potential can then be written as

$$V(H, \Phi) = m_H^2 \operatorname{tr} H^\dagger H + m_\Phi^2 \operatorname{tr} \Phi^2 + \frac{\lambda_H}{2} (\operatorname{tr} H^\dagger H)^2 + \frac{\lambda_\Phi}{2} (\operatorname{tr} \Phi^2)^2 + \lambda_{H\Phi} \operatorname{tr} \Phi^2 \operatorname{tr} H^\dagger H + \cdots. \tag{C.6}$$

Note that there is no trilinear term (in the custodial $\mathrm{SU}(2)$ limit) because H always appears in the singlet combination $\operatorname{tr} H^\dagger H$ due to the antisymmetric property of the $\mathrm{SU}(2)$ ϵ tensor. We take $m_H^2 < 0$ so that electroweak symmetry is broken correctly. Some tuning on the parameters is required to get $v_{\mathrm{EW}} \ll f$ as we do not specify the origin of these parameters.

The degeneracy between $\phi^\pm$ and ϕ^0 will be lifted by the custodial $\mathrm{SU}(2)$ violating effects, including the radiative corrections coming from the $\mathrm{U}(1)_R$ gauge field. The custodial $\mathrm{SU}(2)$ breaking can be parametrized by the spurion matrix $\Xi = \operatorname{diag}(0, 0, 1, -1)$, for instance,

$$f^4 \operatorname{tr} \Sigma \Xi \Sigma^\dagger \Xi = f^4 \left(2 - 2 \frac{\phi^+ \phi^-}{f^2} - \frac{h^\dagger h}{2f^2} + \cdots \right),$$
$$f^4 \operatorname{tr} \Sigma \Xi \Sigma \Xi = f^4 \left(2 - \frac{(\phi^0)^2}{f^2} - \frac{h^\dagger h}{2f^2} + \cdots \right). \tag{C.7}$$

These effects can be thought of as ϕ^0 having a nonzero vacuum expectation value. (Indeed ϕ^0 can have a tadpole term in the presence of custodial $\mathrm{SU}(2)$ breaking.) The possible scalar potential can be obtained by shifting ϕ^0 in eq. (C.6) by a constant. A trilinear term

$$\phi^0 \operatorname{tr} H^\dagger H \tag{C.8}$$

and other terms linear in ϕ^0 are now possible, but will be suppressed if the custodial $\mathrm{SU}(2)$ breaking effects are small.

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